WORKED EXAMPLE

Long-Term Deflection Calculation - Non-Prestressed RC Beam — (EN1992-1-1 2004)

This document presented the worked example of Example 8.4 from Concrete Structures, Stresses and Deformations, Third Edition (Ghali et al, 2006) using EN 1992-1-1: 2004 approach. The results from Case 1 ~ 5 are presented and discussed in the end of the document to give user a clearer overview of the comparison between two approaches.

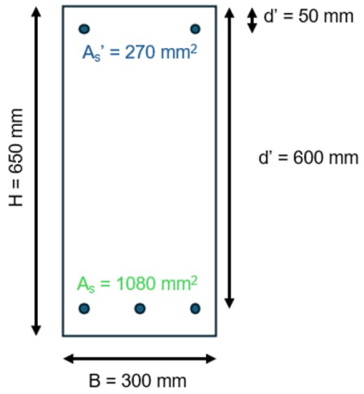
Document ref.	B300×650 — Long-Term Deflection Parameters	Status	Worked calculation (partial)
Design code	EN 1992-1-1: 2004 Cl3.1.4, Cl7.1, Cl7.4.3, Annex B	Revision	1.0

0. Glossary

Symbol	Description
B	Section width
H	Section overall depth
d'	Effective depth to compression steel (top)
d	Effective depth to tension steel (bottom)
A_s'	Compression steel area (top)
A_s	Tension steel area (bottom)
f_{ck}	Characteristic compressive strength
f_{ctm}	Mean tensile strength (EN1992-1-1:2004 Table 3.1)
E_s	Modulus of elasticity of steel
E_{cm}	Modulus of elasticity of concrete
$\varphi(\infty,t_0)$	Creep coefficient – Refer to Appendix A
$\chi(t,t_0)$	Aging coefficient – Not used in EN1992-1-1:2004 Approach.
$E_{cm,eff}$	Effective modulus = $E_{cm}/(1+\varphi)$
$\alpha_e(t_0)$	Modular ratio at (time = t_0) = E_s/E_{cm}
$\alpha_e(t)$	Modular ratio at (time = t) = $E_s/E_{cm,eff}$
$\varepsilon_{cs}(t)$	Free Shrinkage Strain at time = t– Refer to Appendix A

1. Section, Material and Reinforcement Data

The beam is a simply supported, non-prestressed reinforced concrete rectangular section. Section, reinforcement and material parameters are shown in table below:



Example of Case 1 = Beam cross-section: B×H = 300×650 mm, with top and bottom reinforcement.

Case	Section	As' (mm^2)	As (mm^2)	d' (mm)	d (mm)
1	B300x650	270	1080	50	600
2	B300x650	-	3217	50	600
3	B200x650	982	982	57.5	597.5
4	B800x1000	9651	11260	93	907
5	B800x1000	9651	11260	93	907

1.1 Material Properties

Case	f_{ck} (MPa)	f_{ctm} (MPa)	E_{cm} (GPa)	$\varphi(\infty,t_0)$	$\chi(t,t_0)$	$\varepsilon_{cs}(t)$ (10^{-6})	$\alpha_e(t_0)$	$\alpha_e(t)$
1	-	2.50	30	2.5	0.8	-250	6.67	23.33
2	32	3.02	33	2.5	1.0	-485	6.06	21.21
3	32	3.02	33	2.5	1.0	-485	6.06	21.21
4	32	3.02	33	2.5	1.0	-357	6.06	21.21
5	60	4.35	39	2.5	1.0	-357	5.13	17.95

In the following sections, the example calculation for Case 1 will be only presented. Result comparison for Case 1~5 will still be discussed in the "Result and Conclusion" section.

2. Uncracked Section Properties

Concrete area and second moment of area of the gross (uncracked) section:

Transformed uncracked-section neutral-axis depth (from the top fibre) and moment of inertia:

$$x_u = \frac{BH\frac{H}{2} + (\alpha_e - 1)(A_s d + A_s' d')}{BH + (\alpha_e - 1)(A_s + A_s')}$$

$$I_u = \frac{BH^3}{12} + BH\left(\frac{H}{2} - x_u\right)^2 + (\alpha_e - 1)[A_s(d - x_u)^2 + A_s'(x_u - d')^2]$$

$x_u(t_0)$ = 331.23 mm

$x_u(t)$ = 347.10 mm

$I_u(t_0)$ = 7.436 × 10⁹ mm⁴

$I_u(t)$ = 9.036 × 10⁹ mm⁴

3. Cracked Section Properties

Depth of the compression zone x_c, solved from the first-moment quadratic:

$$\frac{1}{2}Bx_c^2 + [\alpha_e A_s + (\alpha_e - 1)A_s']x_c - [\alpha_e A_s d + (\alpha_e - 1)A_s' d'] = 0$$

$$x_c = \frac{-[\alpha_e A_s + (\alpha_e - 1)A_s'] + \sqrt{[\alpha_e A_s + (\alpha_e - 1)A_s']^2 + 2B[\alpha_e A_s d + (\alpha_e - 1)A_s' d']}}{B}$$

$$I_c = \frac{Bx_c^3}{3} + \alpha_e A_s(d - x_c)^2 + (\alpha_e - 1)A_s'(x_c - d')^2$$

$x_c(t_0)$ = 144.56 mm

$x_c(t)$ = 233.02 mm

$I_c(t_0)$ = 1.809 × 10⁹ mm⁴

$I_c(t)$ = 4.861 × 10⁹ mm⁴

4. Cracking Moment

The cracking moment M_cr uses the uncracked section modulus and the concrete tensile strength:

$$M_{cr} = \frac{f_{ctm} I_u(t_0)}{h - x_u(t_0)}$$

M_{cr} = 58.3 kNm

5. Calculation of Curvature

The calculation of curvature follows EN1992-1-1:2004 Cl 7.4.3(6) Eqn. (7.21) for Shrinkage Curvature and Cl 7.4.3(3) Eqn. (7.18).

$$\frac{1}{r_{total}} = \frac{1}{r} + \frac{1}{r_{cs}} = \left[\xi \frac{1}{r_I} + (1-\xi)\frac{1}{r_{II}}\right] + \left[\xi \frac{1}{r_{cs,I}} + (1-\xi)\frac{1}{r_{cs,II}}\right]$$

Where:

$$\frac{1}{r_I} = \frac{M}{E I_u}$$

$$\frac{1}{r_{II}} = \frac{M}{E I_c}$$

$$\frac{1}{r_{cs,I}} = \varepsilon_{cs}\alpha_e \frac{S_u}{I_u}$$

$$\frac{1}{r_{cs,II}} = \varepsilon_{cs}\alpha_e \frac{S_c}{I_c}$$

$$S = \Sigma A_{si}(d_i - x) - \Sigma A_{si}'(x - d_i')$$

$$\xi = 1 - \beta\left(\frac{M_{cr}}{M}\right)^2, if\ M_{cr} < M, else\ \xi = 0$$

Modified Stn	Moment	$1/r_I$	$1/r_{II}$	$1/r_{cs,I}$	$1/r_{cs,II}$	Factor ξ	1/r	$1/r_{cs}$	$1/r_{Total}$
0	0.00 kNm	0.000	0.000	0.125	0.416	0.000	0.000	0.125	0.125
800	48.96 kNm	0.632	1.175	0.125	0.416	0.000	0.632	0.125	0.757
1600	87.04 kNm	1.124	2.089	0.125	0.416	0.776	1.872	0.351	2.223
2400	114.24 kNm	1.475	2.742	0.125	0.416	0.870	2.577	0.378	2.955
3200	130.56 kNm	1.686	3.133	0.125	0.416	0.900	2.989	0.387	3.376
4000	136.00 kNm	1.756	3.264	0.125	0.416	0.908	3.125	0.389	3.515
4800	130.56 kNm	1.686	3.133	0.125	0.416	0.900	2.989	0.387	3.376
5600	114.24 kNm	1.475	2.742	0.125	0.416	0.870	2.577	0.378	2.955
6400	87.04 kNm	1.124	2.089	0.125	0.416	0.776	1.872	0.351	2.223
7200	48.96 kNm	0.632	1.175	0.125	0.416	0.000	0.632	0.125	0.757
8000	0.00 kNm	0.000	0.000	0.125	0.416	0.000	0.000	0.125	0.125

6. Calculation of Deflection

The derivation of deflection calculation is presented in this section. More details may be found in Appendix A below for the visualization of the double integration method for Curvature and Deflection formula.

(1) Curvature & Deflection Formula

$$Curvature\ \theta(x) = \int \frac{M(x)}{EI} dx + C_1$$

$$Deflection\ \Delta(x) = \int \int (\frac{M(x)}{EI} dx + C_1)\ dx$$

$$Deflection\ \Delta(x) = \int \int \frac{M(x)}{EI} dx + C_1 x + C_2$$

(2) Apply Boundary Condition

$$\therefore When\ x = 0,\ C_2 = \Delta_{start,\ x=0}$$

$$\therefore When\ x = L,\ C_1 = \frac{-\int \int \frac{M(L)}{EI} dx + \Delta_{end,\ x=L}}{L}$$

$$Deflection\ \Delta(x) = \int \int \frac{M(x)}{EI} dx + (\frac{-\int_{x=0}^{x=L} \int \frac{M(x)}{EI} dx + \Delta_{end,\ x=L} - \Delta_{start,\ x=0}}{L})x + \Delta_{start,\ x=0}$$

(3) Final Formula of Deflection at Each Point of Beam

$$\therefore Deflection\ \Delta(x) = A + B + C$$

$A = Double\ Integral\ of \frac{M(x)}{EI}$

$B = [Value\ of$ double integral from x = 0 to L + Deflection at beam end (x = L) $-$ $Deflection\ at\ beam\ start\ (x = 0)] * \left(\frac{x}{L}\right)$

$C = Deflection\ at\ beam\ start\ (x = 0)$

Notes:

$$\theta_x = \theta_{x-1} \left(\frac{\frac{1}{r_x} + \frac{1}{r_{x-1}}}{2} \right) * (L_{stn,x} - L_{stn,x-1})$$

$$\Delta_x = \Delta_{x-1} \left(\frac{\theta_x + \theta_{x-1}}{2} \right) * (L_{stn,x} - L_{stn,x-1})$$

$(Correction\ C_1\ \&\ C_2)$ $C_2 = 0\ for\ simply\ supported\ beam,$ $C_1 = \Delta_{\mathrm{End,Uncorrected}} \left(\frac{x}{L}\right)$

$1/r_{Total}$	θ_{Total} (10^{-3} rad)	$\Delta_{Total,U}$ (mm)	Correction	$\Delta_{Total,Final}$ (mm)
0.125	0.000	0.000	0.000	0.000
0.757	0.352	0.141	7.124	-6.983
2.223	1.544	0.900	14.248	-13.348
2.955	3.616	2.964	21.371	-18.407
3.376	6.148	6.869	28.495	-21.626
3.515	8.905	12.891	35.619	-22.728
3.376	11.661	21.117	42.743	-21.626
2.955	14.194	31.459	49.866	-18.407
2.223	16.265	43.642	56.990	-13.348
0.757	17.457	57.131	64.114	-6.983
0.125	17.809	71.238	71.238	0.000

7. Result Comparison

The result comparison between the approaches according to *Concrete Structures, Stresses and Deformations, Third Edition, A Ghali, R Favre and M Elbadry and EN1992-1-1: 2004 are tabulated as shown below:*

Table 1: Case 1 (Example 8.4 from Concrete Structures, Stresses and Deformations, Third Edition, A Ghali, R Favre and M Elbadry, pages 285-289)

CASE AND FEATURE TESTED	TEXTBOOK RESULT	CALCULATED RESULT	DIFFERENCE (%)
Mid-Span Displacement No Creep/Shrinkage (m)	14.4	13.1	9.03
Mid-Span Displacement With Creep/Shrinkage (m)	23.9	22.7	5.02
Note: R300x650, Cover to Rebar Center = 50mm, As = 1080 mm^2, As' = 270 mm^2, E_{cm} = 30 GPa, Aging Coefficient X = 0.8, $\phi(\infty,t_0)$ = 2.5, $\varepsilon cs(t) = -250x10^{-6}$, Aging Coefficient X only used in *Concrete Structures, Stresses and Deformations, Third Edition (Ghali et al, 2006)*			

Table 2: Case 2 = B300x650, fck = 32 MPa, $\varphi(\infty,t_0)$ = 2.5, $\varepsilon_{cs}(t) = -485\ x10^{-6}$

CASE AND FEATURE TESTED	TEXTBOOK RESULT	CALCULATED RESULT	ABSOLUTE DIFFERENCE (mm)	DIFFERENCE (%)
Mid-Span Displacement With Creep/Shrinkage				
Line Load = 5 kN/m	6.47	5.64	0.83	12.9
Line Load = 10 kN/m	11.26	10.60	0.66	5.9
Line Load = 15 kN/m	15.53	15.64	0.11	0.7
Line Load = 20 kN/m	19.31	19.72	0.41	2.1
Line Load = 25 kN/m	22.95	23.35	0.40	1.7
Line Load = 30 kN/m	26.52	26.89	0.37	1.4
Line Load = 40 kN/m	33.59	33.73	0.14	0.4
Line Load = 50 kN/m	40.60	40.47	0.14	0.3
Line Load = 60 kN/m	47.60	47.17	0.43	0.9
Line Load = 70 kN/m	54.58	53.84	0.74	1.4
Line Load = 80 kN/m	61.56	60.51	1.05	1.7
Line Load = 90 kN/m	68.53	67.16	1.36	2.0
Line Load = 100 kN/m	75.50	73.82	1.68	2.2

Table 3: Case 3 = B200x650, fck = 32 MPa, $\varphi(\infty,t_0)$ = 2.5, $\varepsilon_{cs}(t) = -485\ x10^{-6}$

CASE AND FEATURE TESTED	TEXTBOOK RESULT	CALCULATED RESULT	ABSOLUTE DIFFERENCE (mm)	DIFFERENCE (%)
Mid-Span Displacement With Creep/Shrinkage				
Line Load = 5 kN/m	1.23	1.21	0.02	1.3
Line Load = 10 kN/m	2.46	2.43	0.03	1.3
Line Load = 15 kN/m	7.19	6.77	0.41	5.8
Line Load = 20 kN/m	9.83	9.90	0.07	0.7
Line Load = 25 kN/m	12.27	12.44	0.17	1.4
Line Load = 30 kN/m	14.61	14.85	0.23	1.6
Line Load = 40 kN/m	19.17	19.85	0.68	3.5
Line Load = 50 kN/m	23.66	24.50	0.84	3.6
Line Load = 60 kN/m	28.12	29.10	0.98	3.5
Line Load = 70 kN/m	32.55	33.67	1.11	3.4
Line Load = 80 kN/m	36.98	38.21	1.23	3.3
Line Load = 90 kN/m	41.40	42.75	1.35	3.3
Line Load = 100 kN/m	45.82	47.28	1.46	3.2

Table 4: Case 4 = B800x1000, fck = 32 MPa, $\varphi(\infty,t_0) = 2.5$, $\varepsilon_{cs}(t) = -357 \times 10^{-6}$

CASE AND FEATURE TESTED	TEXTBOOK RESULT	CALCULATED RESULT	ABSOLUTE DIFFERENCE (mm)	DIFFERENCE (%)
Mid-Span Displacement With Creep/Shrinkage				
Line Load = 5 kN/m	1.43	1.46	0.03	1.8
Line Load = 10 kN/m	2.51	2.50	0.01	0.4
Line Load = 15 kN/m	3.59	3.54	0.05	1.3
Line Load = 20 kN/m	4.67	4.59	0.08	1.8
Line Load = 25 kN/m	5.75	5.20	0.55	9.6
Line Load = 30 kN/m	9.76	10.01	0.25	2.6
Line Load = 40 kN/m	13.75	14.12	0.37	2.7
Line Load = 50 kN/m	17.25	17.53	0.28	1.6
Line Load = 60 kN/m	20.55	20.75	0.20	1.0
Line Load = 70 kN/m	23.75	24.18	0.43	1.8
Line Load = 80 kN/m	26.88	27.32	0.44	1.6
Line Load = 90 kN/m	29.99	30.42	0.44	1.5
Line Load = 100 kN/m	33.06	33.49	0.43	1.3
Line Load = 110 kN/m	36.13	36.55	0.42	1.2
Line Load = 120 kN/m	39.18	39.58	0.40	1.0
Line Load = 130 kN/m	42.22	42.61	0.39	0.9
Line Load = 140 kN/m	45.26	45.63	0.37	0.8
Line Load = 150 kN/m	48.29	48.65	0.35	0.7
Line Load = 160 kN/m	51.32	51.65	0.34	0.7
Line Load = 170 kN/m	54.34	54.66	0.32	0.6
Line Load = 180 kN/m	57.37	57.66	0.30	0.5
Line Load = 190 kN/m	60.39	60.66	0.28	0.5
Line Load = 200 kN/m	63.40	63.66	0.26	0.4

Table 5: Case 5 = B800x1000, fck = 60 MPa, $\varphi(\infty,t_0) = 2.5$, $\varepsilon_{cs}(t) = -357 \times 10^{-6}$

CASE AND FEATURE TESTED	TEXTBOOK RESULT	CALCULATED RESULT	ABSOLUTE DIFFERENCE (mm)	DIFFERENCE (%)
Mid-Span Displacement With Creep/Shrinkage				
Line Load = 5 kN/m	1.32	1.34	0.03	2.1
Line Load = 10 kN/m	2.31	2.31	0.01	0.2
Line Load = 15 kN/m	3.31	3.27	0.04	1.2
Line Load = 20 kN/m	4.31	4.24	0.07	1.6
Line Load = 25 kN/m	5.30	5.20	0.10	1.9
Line Load = 30 kN/m	6.30	6.16	0.14	2.2
Line Load = 40 kN/m	8.29	8.91	0.62	7.5
Line Load = 50 kN/m	15.83	14.54	1.29	8.1
Line Load = 60 kN/m	19.28	17.85	1.43	7.4
Line Load = 70 kN/m	22.55	21.88	0.66	2.9
Line Load = 80 kN/m	25.71	25.11	0.60	2.4
Line Load = 90 kN/m	28.81	28.24	0.58	2.0
Line Load = 100 kN/m	31.87	31.30	0.57	1.8
Line Load = 110 kN/m	34.90	34.68	0.22	0.6
Line Load = 120 kN/m	37.92	37.75	0.17	0.4
Line Load = 130 kN/m	40.91	40.78	0.13	0.3
Line Load = 140 kN/m	43.89	43.80	0.10	0.2
Line Load = 150 kN/m	46.87	46.80	0.07	0.2
Line Load = 160 kN/m	49.84	49.78	0.06	0.1
Line Load = 170 kN/m	52.80	52.75	0.05	0.1
Line Load = 180 kN/m	55.76	55.72	0.04	0.1
Line Load = 190 kN/m	58.71	58.67	0.04	0.1
Line Load = 200 kN/m	61.66	61.62	0.04	0.1

8. Conclusion

The results for Case 1 - Worked Example 8.4 from Concrete Structures, Stresses and Deformations, Third Edition (Ghali et al, 2006) and Case 2~5 using EN 1992-1-1: 2004 approach are presented in this example calculation technical note. In summary, we noted that although the maximum difference % between two approaches is as high as 12.9%, but the absolute difference of maximum deflection value is limited to +/- 2.0 mm (to be safe).

There are several reasons identified that may cause the deviation between two approaches. They are, but not limited to, deviation due to round off, deviation due to the presence of aging coefficient (not appear in EN 1992-1-1: 2004), and deviation due to double integration approaches (finer station span provide better approximation towards correct value).

APPENDIX A:
GENERAL CALCULATION PROCEDURE

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General Calculation Procedure

General Calculation Procedure

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1. Calculation of Uncracked and Cracked Second Moment of Inertia.
2. Calculation of Cracking Moment
3. Calculation of Creep Coefficient
4. Calculation of Shrinkage Strain and Curvature
5. Derivation of Deflection from Respective Curvature

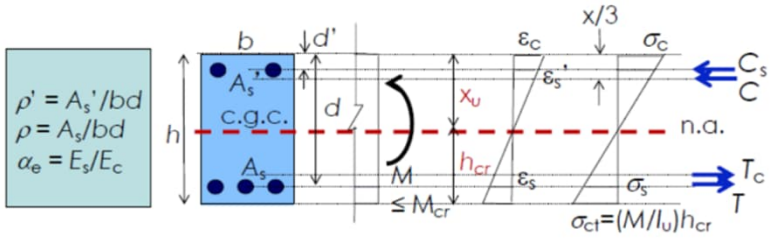
Calculation of Second Moment of Inertia I_u & I_c

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3.2.1 Mechanism of flexural cracking

Before cracking (uncracked section):

Refer to Ch2, Slide 18-19



(Step 1a)
Calculate Uncracked NA Depth x_u & $I_{cracked}$ Equation

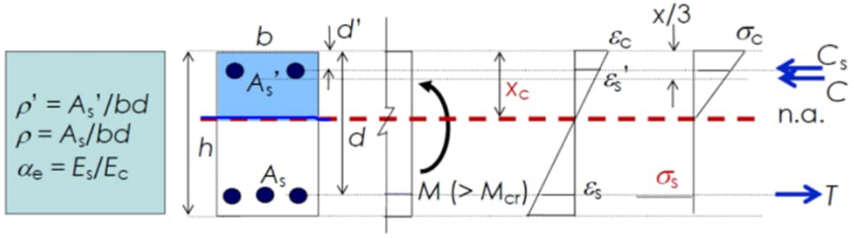
$$x_u = \frac{\frac{bh^2}{2} + (\alpha_e - 1)(A_s d + A_s' d')}{bh + (\alpha_e - 1)(A_s + A_s')}$$

$$I_{uncracked} = \frac{bh^3}{12} + bh\left(\frac{h}{2} - x_u\right)^2 + (\alpha_e - 1)[A_s(d - x_u)^2 + A_s'(x_u - d)^2]$$

3.2.1 Mechanism of flexural cracking

After cracking (cracked section):

Refer to Ch2, Slide 24-25



(Step 1b)
Calculate Cracked NA Depth x_u & $I_{cracked}$ Equation

$$x_c = \frac{\frac{bx_c^2}{2} + \alpha_e A_s d + (\alpha_e - 1)A_s' d'}{bx_c + \alpha_e A_s + (\alpha_e - 1)A_s'}$$

$$\frac{1}{2}bx_c^2 + [\alpha_e A_s + (\alpha_e - 1)A_s']x_c - [\alpha_e A_s d + (\alpha_e - 1)A_s' d'] = 0$$

$$x_c = \frac{-[\alpha_e A_s + (\alpha_e - 1)A_s'] \pm \sqrt{[\alpha_e A_s + (\alpha_e - 1)A_s']^2 + 2b[\alpha_e A_s d + (\alpha_e - 1)A_s' d']}}{b}$$

$$I_{cracked} = \frac{bx_c^3}{3} + \alpha_e A_s(d - x_c)^2 + (\alpha_e - 1)A_s'(x_c - d')^2$$

Quadratic Equation

$$x_c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Calculation of Cracking Moment M_{cr}

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SECTION 7 SERVICEABILITY LIMIT STATES (SLS)

7.1 General

(2) In the calculation of stresses and deflections, cross-sections should be assumed to be uncracked provided that the flexural tensile stress does not exceed $f_{ct,eff}$. The value of $f_{ct,eff}$ may be taken as f_{ctm} or $f_{ctm,fl}$ provided that the calculation for minimum tension reinforcement is also based on the same value. For the purposes of calculating crack widths and tension stiffening f_{ctm} should be used.

7.4.3 Checking deflections by calculation

(4) Deformations due to loading may be assessed using the tensile strength and the effective modulus of elasticity of the concrete (see (5)).

Table 3.1 indicates the range of likely values for tensile strength. In general, the best estimate of the behaviour will be obtained if f_{ctm} is used. Where it can be shown that there are no axial tensile stresses (e.g. those caused by shrinkage or thermal effects) the flexural tensile strength, $f_{ctm,fl}$, (see 3.1.8) may be used.

(Step 1c)
Calculate Cracking Moment

$$M_{cr} = \frac{f_{ctm} I_u}{h - x_u}$$

Calculation of Creep Coefficient $\varphi(\infty,t_0)$

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SECTION 7 SERVICEABILITY LIMIT STATES (SLS)

7.4.3 Checking deflections by calculation

(5) For loads with a duration causing creep, the total deformation including creep may be calculated by using an effective modulus of elasticity for concrete according to Expression (7.20):

$$E_{c,eff} = \frac{E_{cm}}{1+\varphi(\infty,t_0)} \quad (7.20)$$

← Unknown

where:

[AC1] $\varphi(\infty,t_0)$ is the creep coefficient relevant for the load and time interval (see 3.1.4) [AC1]

3.1.4 Creep and shrinkage

(1)P Creep and shrinkage of the concrete depend on the ambient humidity, the dimensions of the element and the composition of the concrete. Creep is also influenced by the maturity of the concrete when the load is first applied and depends on the duration and magnitude of the loading.

Skipped*

(2) The creep coefficient, $\varphi(t,t_0)$, is related to E_c, the tangent modulus, which may be taken as 1,05 E_{cm}. Where great accuracy is not required, the value found from Figure 3.1 may be considered as the creep coefficient, provided that the concrete is not subjected to a compressive stress greater than 0,45 f_{ck} (t_0) at an age t_0, the age of concrete at the time of loading.

Unknown

Note: For further information, including the development of creep with time, Annex B may be used.

Calculation of Creep Coefficient $\varphi(\infty,t_0)$

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EN1992-1-1: 2004 Annex B (Pg. 1)

ANNEX B (Informative)

Creep and shrinkage strain

B.1 Basic equations for determining the creep coefficient

(1) The creep coefficient $\varphi(t,t_0)$ may be calculated from:

$$\varphi(t,t_0) = \varphi_0 \cdot \beta_c(t,t_0) \quad \text{(B.1)}$$

where:

φ_0 is the notional creep coefficient and may be estimated from:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) \quad \text{(B.2)}$$

φ_{RH} is a factor to allow for the effect of relative humidity on the notional creep coefficient:

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0{,}1 \cdot \sqrt[3]{h_0}} \quad \text{for } f_{cm} \le 35 \text{ MPa} \quad \text{(B.3a)}$$

$$\varphi_{RH} = \left[1 + \frac{1 - RH/100}{0{,}1 \cdot \sqrt[3]{h_0}} \cdot \alpha_1\right] \cdot \alpha_2 \quad \text{for } f_{cm} > 35 \text{ MPa} \quad \text{(B.3b)}$$

RH is the relative humidity of the ambient environment in %

$\beta(f_{cm})$ is a factor to allow for the effect of concrete strength on the notional creep coefficient:

$$\beta(f_{cm}) = \frac{16{,}8}{\sqrt{f_{cm}}} \quad \text{(B.4)}$$

f_{cm} is the mean compressive strength of concrete in MPa at the age of 28 days

$\beta(t_0)$ is a factor to allow for the effect of concrete age at loading on the notional creep coefficient:

$$\beta(t_0) = \frac{1}{(0{,}1 + t_0^{0{,}20})} \quad \text{(B.5)}$$

h_0 is the notional size of the member in mm where:

$$h_0 = \frac{2A_c}{u} \quad \text{(B.6)}$$

A_c is the cross-sectional area

u is the perimeter of the member in contact with the atmosphere

$\beta_c(t,t_0)$ is a coefficient to describe the development of creep with time after loading, and may be estimated using the following Expression:

$$\beta_c(t,t_0) = \left[\frac{(t - t_0)}{(\beta_H + t - t_0)}\right]^{0{,}3} \quad \text{(B.7)}$$

t is the age of concrete in days at the moment considered

t_0 is the age of concrete at loading in days

$t - t_0$ is the non-adjusted duration of loading in days

β_H is a coefficient depending on the relative humidity (RH in %) and the notional member size (h_0 in mm). It may be estimated from:

$$\beta_H = 1{,}5\,[1 + (0{,}012\,RH)^{18}]\,h_0 + 250 \le 1500 \quad \text{for } f_{cm} \le 35 \quad \text{(B.8a)}$$

$$\beta_H = 1{,}5\,[1 + (0{,}012\,RH)^{18}]\,h_0 + 250\,\alpha_3 \le 1500\,\alpha_3 \quad \text{for } f_{cm} \ge 35 \quad \text{(B.8b)}$$

$\alpha_{1/2/3}$ are coefficients to consider the influence of the concrete strength:

$$\alpha_1 = \left[\frac{35}{f_{cm}}\right]^{0{,}7} \quad \alpha_2 = \left[\frac{35}{f_{cm}}\right]^{0{,}2} \quad \alpha_3 = \left[\frac{35}{f_{cm}}\right]^{0{,}5} \quad \text{(B.8c)}$$

(2) The effect of type of cement (see 3.1.2 (6)) on the creep coefficient of concrete may be taken into account by modifying the age of loading t_0 in Expression (B.5) according to the following Expression:

$$t_0 = t_{0,T} \cdot \left(\frac{9}{2 + t_{0,T}^{1,2}} + 1\right)^{\alpha} \ge 0{,}5 \quad \text{(B.9)}$$

where:

$t_{0,T}$ is the temperature adjusted age of concrete at loading in days adjusted according to Expression (B.10)

α is a power which depends on type of cement
= -1 for cement Class S
= 0 for cement Class N
= 1 for cement Class R

(3) The effect of elevated or reduced temperatures within the range 0 – 80°C on the maturity of concrete may be taken into account by adjusting the concrete age according to the following Expression:

$$t_T = \sum_{i=1}^{n} e^{-(4000/[273 + T(\Delta t_i)] - 13{,}65)} \cdot \Delta t_i \quad \text{(B.10)}$$

where:

t_T is the temperature adjusted concrete age which replaces t in the corresponding equations

$T(\Delta t_i)$ is the temperature in °C during the time period Δt_i

Δt_i is the number of days where a temperature T prevails.

The mean coefficient of variation of the above predicted creep data, deduced from a computerised data bank of laboratory test results, is of the order of 20%.

The values of $\varphi(t,t_0)$ given above should be associated with the tangent modulus E_c. When a less accurate estimate is considered satisfactory, the values given in Figure 3.1 of 3.1.4 may be adopted for creep of concrete at 70 years.

Calculation of Shrinkage Strain and Curvature

(Member of Surbana Jurong Group)

7.4.3 Checking deflections by calculation

(6) Shrinkage curvatures may be assessed using Expression (7.21):

$$\frac{1}{r_{cs}} = \varepsilon_{cs}\alpha_e \frac{S}{I} \qquad (7.21)$$

Unknown

where:

$1/r_{cs}$ is the curvature due to shrinkage

ε_{cs} is the free shrinkage strain (see 3.1.4) — Note*

S is the first moment of area of the reinforcement about the centroid of the section

I is the second moment of area of the section

α_e is the effective modular ratio

$\alpha_e = E_s / E_{c,eff}$

S and I should be calculated for the uncracked condition and the fully cracked condition, the final curvature being assessed by use of Expression (7.18).

3.1.4 Creep and shrinkage

(6) The total shrinkage strain is composed of two components, the drying shrinkage strain and the autogenous shrinkage strain. The drying shrinkage strain develops slowly, since it is a function of the migration of the water through the hardened concrete. The autogenous shrinkage strain develops during hardening of the concrete: the major part therefore develops in the early days after casting. Autogenous shrinkage is a linear function of the concrete strength. It should be considered specifically when new concrete is cast against hardened concrete. Hence the values of the total shrinkage strain ε_{cs} follow from

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca} \qquad (3.8)$$

where:

ε_{cs} is the total shrinkage strain

ε_{cd} is the drying shrinkage strain

ε_{ca} is the autogenous shrinkage strain

Unknown

The final value of the drying shrinkage strain, $\varepsilon_{cd,\infty}$ is equal to $k_h \cdot \varepsilon_{cd,0}$. $\varepsilon_{cd,0}$ may be taken from Table 3.2 (expected mean values, with a coefficient of variation of about 30%).

Note: The formula for $\varepsilon_{cd,0}$ is given in Annex B.

3.1.4 Creep and shrinkage

The development of the drying shrinkage strain in time follows from:

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} \qquad (3.9)$$

Unknown

where

k_h is a coefficient depending on the notional size h_0 according to Table 3.3

Table 3.3 Values for k_h in Expression (3.9)

h_0	k_h
100	1.0
200	0.85
300	0.75
≥ 500	0.70

$$\beta_{ds}(t,t_s) = \frac{(t-t_s)}{(t-t_s)+0{,}04\sqrt{h_0^3}} \qquad (3.10)$$

where:

t is the age of the concrete at the moment considered, in days

t_s is the age of the concrete (days) at the beginning of drying shrinkage (or swelling). Normally this is at the end of curing.

h_0 is the notional size (mm) of the cross-section
$= 2A_c/u$

where:

A_c is the concrete cross-sectional area

u is the perimeter of that part of the cross section which is exposed to drying

The autogenous shrinkage strain follows from:

$$\varepsilon_{ca}(t) = \beta_{as}(t)\,\varepsilon_{ca}(\infty) \qquad (3.11)$$

where:

$$\varepsilon_{ca}(\infty) = 2{,}5\,(f_{ck} - 10)\,10^{-6} \qquad (3.12)$$

and

$$\beta_{as}(t) = 1 - \exp(-0{,}2t^{0,5}) \qquad (3.13)$$

where t is given in days.

Calculation of Shrinkage Strain and Curvature

(Member of Surbana Jurong Group)

EN1992-1-1: 2004 Annex B (Pg. 2)

B.2 Basic equations for determining the drying shrinkage strain

(1) The basic drying shrinkage strain $\varepsilon_{cd,0}$ is calculated from

$$\varepsilon_{cd,0} = 0{,}85\left[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right] \cdot 10^{-6} \cdot \beta_{RH} \qquad \text{(B.11)}$$

$$\beta_{RH} = 1{,}55\left[1 - \left(\frac{RH}{RH_0}\right)^3\right] \qquad \text{(B.12)}$$

where:

f_{cm} is the mean compressive strength (MPa)
f_{cmo} = 10 Mpa
α_{ds1} is a coefficient which depends on the type of cement (see 3.1.2 (6))
= 3 for cement Class *S*
= 4 for cement Class *N*
= 6 for cement Class *R*
α_{ds2} is a coefficient which depends on the type of cement
= 0,13 for cement Class *S*
= 0,12 for cement Class *N*
= 0,11 for cement Class *R*
RH is the ambient relative humidity (%)
RH_0 = 100%.

Note: exp{ } has the same meaning as $e^{( )}$

Derivation of Deflection from Respective Curvature

K T P
(Member of Surbana Jurong Group)

EN1992-1-1: 2004 Cl 7.4.3

(3) Members which are not expected to be loaded above the level which would cause the tensile strength of the concrete to be exceeded anywhere within the member should be considered to be uncracked. Members which are expected to crack, but may not be fully cracked, will behave in a manner intermediate between the uncracked and fully cracked conditions and, for members subjected mainly to flexure, an adequate prediction of behaviour is given by Expression (7.18):

$$\alpha = \zeta\alpha_{II} + (1 - \zeta)\alpha_{I} \qquad (7.18)$$

where

α is the deformation parameter considered which may be, for example, a strain, a curvature, or a rotation. (As a simplification, α may also be taken as a deflection - see (6) below)

α_I, α_{II} are the values of the parameter calculated for the uncracked and fully cracked conditions respectively

ζ is a distribution coefficient (allowing for tensioning stiffening at a section) given by Expression (7.19):

$$\zeta = 1 - \beta\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2 \qquad (7.19)$$

ζ = 0 for uncracked sections

β is a coefficient taking account of the influence of the duration of the loading or of repeated loading on the average strain

= 1,0 for a single short-term loading

= 0,5 for sustained loads or many cycles of repeated loading

σ_s is the stress in the tension reinforcement calculated on the basis of a cracked section

σ_{sr} is the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking

Note: σ_{sr}/σ_s may be replaced by M_{cr}/M for flexure or N_{cr}/N for pure tension, where M_{cr} is the cracking moment and N_{cr} is the cracking force.

(7) The most rigorous method of assessing deflections using the method given in (3) above is to compute the curvatures at frequent sections along the member and then calculate the deflection by numerical integration. In most cases it will be acceptable to compute the deflection twice, assuming the whole member to be in the uncracked and fully cracked condition in turn, and then interpolate using Expression (7.18).

Note: Where simplified methods of calculating deflections are used they should be based on the properties given in this Standard and substantiated by tests.

1

$$\frac{1}{r_{total}} = \left[\xi\frac{1}{r_{Cracked}} + (1-\xi)\frac{1}{r_{uncracked}}\right] + \left[\xi\frac{1}{r_{cs,II}} + (1-\xi)\frac{1}{r_{cs,I}}\right]$$

Elastic Curvature ***Shrinkage Curvature***

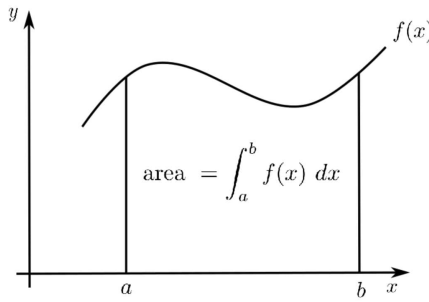
2

Curvature & Deflection Formula

$$Curvature\ \theta(x) = \int \frac{M(x)}{EI}dx + C_1$$

$$Deflection\ \Delta(x) = \int\int \left(\frac{M(x)}{EI}dx + C_1\right)dx$$

$$Deflection\ \Delta(x) = \int\int \frac{M(x)}{EI}dx + C_1x + C_2$$



Apply Boundary Condition

$$\therefore When\ x = 0, C_2 = \Delta_{start,x=0}$$

$$\therefore When\ x = L, C_1 = \frac{-\int\int \frac{M(L)}{EI}dx * + \Delta_{end,x=L}}{L}$$

$$Deflection\ \Delta(x) = \int\int \frac{M(x)}{EI}dx + \left(\frac{-\int_{x=0}^{x=L}\int \frac{M(x)}{EI}dx + \Delta_{end,x=L} - \Delta_{start,x=0}}{L}\right)x + \Delta_{start,x=0}$$

#Careful with the sign convention of the deflection

Final Formula of Deflection at Each Point of Beam

$$\therefore Deflection\ \Delta(x) = A + B + C$$

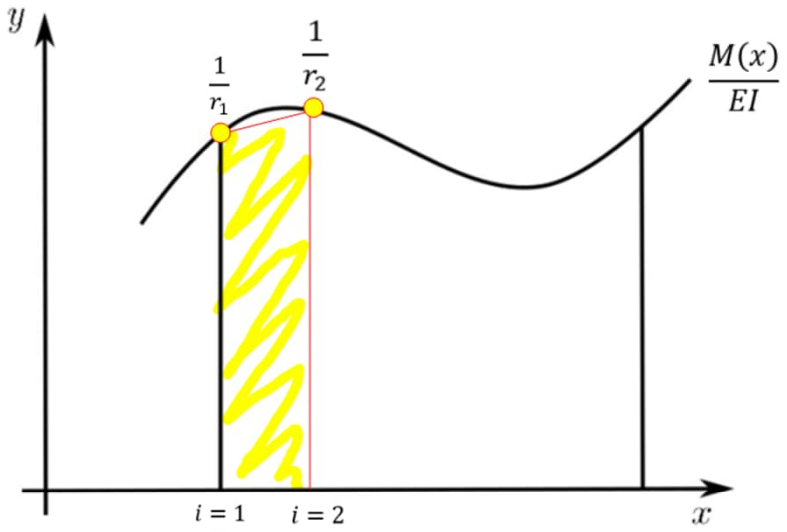
$$A = Double\ Integral\ of\ \frac{M(x)}{EI}$$

$$B = [Value\ of\ \text{double integral from x = 0 to L + Deflection at beam end (x = L)} - Deflection\ at\ beam\ start\ (x = 0)] * \left(\frac{x}{L}\right)$$

$$C = Deflection\ at\ beam\ start\ (x = 0)$$

Uncorrected Deflection Calculation

(Member of Surbana Jurong Group)



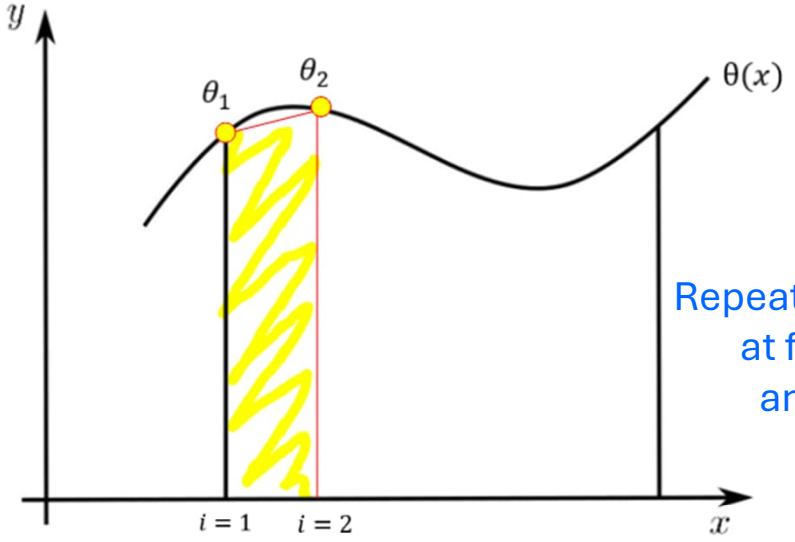
$$\theta_1 = \theta_0 + \left(\frac{\frac{1}{r_1} + \frac{1}{r_0}}{2}\right)$$

$$\theta_2 = \theta_1 + \left(\frac{\frac{1}{r_2} + \frac{1}{r_1}}{2}\right)$$

Shaded Area

...

$$\theta_x = \theta_{x-1} + \left(\frac{\frac{1}{r_x} + \frac{1}{r_{x-1}}}{2}\right)$$



$$\Delta_1 = \Delta_0 + \left(\frac{\theta_1 + \theta_0}{2}\right)$$

$$\Delta_2 = \Delta_1 + \left(\frac{\theta_2 + \theta_1}{2}\right)$$

Shaded Area

...

$$\Delta_{x_n} = \Delta_{x_{n-1}} + \left(\frac{\theta_{x_n} + \theta_{x_{n-1}}}{2}\right)$$

Curvature & Deflection Formula

$$\text{Curvature } \theta(x) = \int \frac{M(x)}{EI}\,dx + C_1$$

$$\text{Deflection } \Delta(x) = \int \int \left(\frac{M(x)}{EI}\,dx + C_1\right) dx$$

$$\text{Deflection } \Delta(x) = \int \int \frac{M(x)}{EI}\,dx + C_1 x + C_2$$

Repeat the calculations at frequent intervals and integrate twice

Correction for Deflection Calculation

(Member of Surbana Jurong Group)

Curvature & Deflection Formula

$$Curvature\ \theta(x) = \int \frac{M(x)}{EI} dx + C_1$$

$$Deflection\ \Delta(x) = \int \int (\frac{M(x)}{EI} dx + C_1)\, dx$$

$$Deflection\ \Delta(x) = \int \int \frac{M(x)}{EI} dx + C_1 x + C_2$$

Integrate twice to obtain the overall deflection

Apply Boundary Condition

$$\therefore When\ x = 0, C_2 = \Delta_{start,x=0}$$

$$\therefore When\ x = L, C_1 = \frac{-\int \int \frac{M(L)}{EI} dx * + \Delta_{end,x=L}}{L}$$

$$Deflection\ \Delta(x) = \int \int \frac{M(x)}{EI} dx + (\frac{-\int_{x=0}^{x=L} \int \frac{M(x)}{EI} dx + \Delta_{end,x=L} - \Delta_{start,x=0}}{L})x + \Delta_{start,x=0}$$

Correction
= C_1 & C_2 for Deflection Integration

Final Formula of Deflection at Each Point of Beam

$$\therefore Deflection\ \Delta(x) = A + B + C$$

$A =$ *Double Integral of* $\frac{M(x)}{EI}$

$B =$ [*Value of* double integral from x = 0 to L + Deflection at beam end (x = L) $-$ *Deflection at beam start* $(x = 0)$] $* \left(\frac{x}{L}\right)$

$C =$ *Deflection at beam start* $(x = 0)$