TECHNICAL NOTE

Combined Shear + Torsion: Derivation of the Design Strut Angle

Closed-form solution of the EC2 crushing-interaction equation for the variable strut angle θ

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Design code	BS EN 1992-1-1 (EC2)	Revision	1.0
Applies to	RC beam/column sections under V + T	Date	2026-06-14

1. Purpose and Scope

This note documents the theoretical basis of the routine that determines the design strut angle θ for reinforced-concrete sections subjected to combined shear and torsion. It records the derivation from first principles, the reduction to a quadratic in cot θ, the closed-form solution actually implemented in code, and the feasibility limits that govern a pass/fail outcome.

The intended audience is engineers reviewing or maintaining the design software. It explains why the implemented formulae take the form they do, so that results can be checked by hand and the code can be safely modified.

Scope: *the derivation covers the concrete strut-crushing check (the upper-bound interaction). Detailing of links and longitudinal steel, once a valid θ is found, is handled separately and is outside this note.*

2. Notation

Symbol	Description	Unit
V_Ed	Design shear force (per direction, major or minor)	kN
T_Ed	Design torsional moment	kNm
V_Rd,c / T_Rd,c	Shear / torsional cracking resistance without reinforcement	kN / kNm
V_Rd,max / T_Rd,max	Maximum shear / torsion resistance governed by strut crushing (θ-dependent)	kN / kNm
θ	Angle of the concrete compression strut (21.8° ≤ θ ≤ 45°)	deg
α_cw	Coefficient for stress state in the compression chord (= 1.0 non-prestressed)	–
ν_1	Strength-reduction factor for concrete cracked in shear = 0.6(1 − f_ck/250)	–
f_cd	Design compressive strength of concrete = f_ck / 1.5	N/mm²
b_w , z	Web width and lever arm (z ≈ 0.9 d)	mm
A_k	Area enclosed by the centre-lines of the connecting walls	mm²

Symbol	Description	Unit
t_ef	Effective wall thickness of the equivalent thin-walled section	mm
A, B, K, C	Derived grouping terms defined in §5–6	–

3. Design Basis (EC2)

The procedure follows the EC2 variable-strut-inclination method. Two resistances are angle-dependent and define the crushing limit of the concrete:

$$V_{Rd,max} = \frac{\alpha_{cw}\, b_w\, z\, \nu_1\, f_{cd}\, \cot\theta}{1 + \cot^2\theta}$$

EC2 Eq. (6.9) — maximum shear resistance.

$$T_{Rd,max} = 2\,\nu_1\,\alpha_{cw}\,f_{cd}\,A_k\,t_{ef}\sin\theta\cos\theta$$

EC2 Eq. (6.30) — maximum torsional resistance. Note v_1 appears in BOTH resistances.

Important: The factor v_1 must appear in the denominator of the torsion term as well as the shear term. Omitting it from the torsion resistance (a common error in summary slides) overestimates the demand ratio and makes the section appear to fail prematurely.

4. Derivation

Step 1 — Interaction trigger

Minimum reinforcement is sufficient only where the un-reinforced interaction is satisfied. Where the following is exceeded, the section must be designed for the combined action and a strut angle solved:

$$\frac{V_{Ed}}{V_{Rd,c}} + \frac{T_{Ed}}{T_{Rd,c}} > 1.0$$

Step 2 — Unity (full-utilisation) equation

At the limit of the crushing check, the combined demand exactly equals the combined θ-dependent resistance:

$$\frac{V_{Ed}}{V_{Rd,max,\theta}} + \frac{T_{Ed}}{T_{Rd,max,\theta}} = 1$$

Substituting the expressions for V_Rd,max and T_Rd,max from §3 and inverting the first fraction (dividing by a fraction = multiplying by its reciprocal):

$$\frac{V_{Ed}\,(1+\cot^2\theta)}{\alpha_{cw}\,b_w\,z\,\nu_1\,f_{cd}\,\cot\theta}+\frac{T_{Ed}}{2\,\nu_1\,\alpha_{cw}\,f_{cd}\,A_k\,t_{ef}\,\sin\theta\cos\theta}=1$$

Step 3 — Trigonometric substitution

The torsion term is converted to the same cot θ basis as the shear term using the identity (since 1/sin²θ = 1 + cot²θ):

$$\sin\theta\cos\theta=\frac{\cot\theta}{1+\cot^2\theta}$$

Applying it to the torsion term gives a common factor of (1 + cot²θ) in both terms:

$$\frac{V_{Ed}\,(1+\cot^2\theta)}{\alpha_{cw}\,b_w\,z\,\nu_1\,f_{cd}\,\cot\theta}+\frac{T_{Ed}\,(1+\cot^2\theta)}{2\,\nu_1\,\alpha_{cw}\,f_{cd}\,A_k\,t_{ef}\,\cot\theta}=1$$

Step 4 — Factor the common θ-term

Both terms now share the factor (1 + cot²θ)/cot θ. Pulling it out leaves a bracket containing only known quantities — no θ:

$$\frac{1+\cot^2\theta}{\cot\theta}\left[\frac{V_{Ed}}{\alpha_{cw}\,b_w\,z\,\nu_1\,f_{cd}}+\frac{T_{Ed}}{2\,\nu_1\,\alpha_{cw}\,f_{cd}\,A_k\,t_{ef}}\right]=1$$

The two terms inside the bracket are defined as A (shear) and B (torsion); their sum is K:

$$A=\frac{V_{Ed}}{\alpha_{cw}\,b_w\,z\,\nu_1\,f_{cd}},\quad B=\frac{T_{Ed}}{2\,\nu_1\,\alpha_{cw}\,f_{cd}\,A_k\,t_{ef}},\quad K=A+B$$

5. Reduction to a Quadratic

With the bracket replaced by K, the unity equation collapses to:

$$K\,(1+\cot^2\theta)=\cot\theta$$

Rearranging into standard quadratic form, with x = cot θ:

$$K\cot^2\theta-\cot\theta+K=0$$

Applying the quadratic formula (a = K, b = −1, c = K):

$$\cot\theta = \frac{1 \pm \sqrt{1 - 4K^2}}{2K}$$

Expressing in terms of C = 1/K

The implementation works with the reciprocal C = 1/(A + B), which maps directly to the spreadsheet column of the same name:

$$C = \frac{1}{K} = \frac{1}{A + B}$$

Substituting K = 1/C, the two roots become:

$$\cot\theta = \frac{C \pm \sqrt{C^2 - 4}}{2}$$

Each root is a value of cot θ; the corresponding angle is recovered by:

$$\theta = \tan^{-1}\left(\frac{1}{\cot\theta}\right)$$

6. Solution Selection and Limits

The quadratic yields two roots. A useful property is that their product equals c/a = 1, so the two roots are reciprocals and the two candidate angles are complementary:

$$\cot\theta_1 \cdot \cot\theta_2 = 1 \quad \Longrightarrow \quad \theta_1 + \theta_2 = 90^\circ$$

The design routine evaluates both roots for each shear direction (major and minor), takes the minimum angle of each pair, then adopts the larger (more onerous) of the two directional results. The adopted angle is floored at 21.8° (the EC2 limit cot θ ≤ 2.5) and the section is rejected if it exceeds 45°.

Feasibility condition

A real solution exists only when the discriminant is non-negative:

$$C^2 - 4 \geq 0 \quad \Longrightarrow \quad C \geq 2 \quad \Longleftrightarrow \quad K \leq 0.5$$

The boundary case sits exactly at 45°:

$$C = 2 \;\Rightarrow\; \cot\theta = 1 \;\Rightarrow\; \theta = 45^\circ$$

Therefore **C < 2** (equivalently K > 0.5) means no valid strut angle exists. Physically the concrete strut crushes before the reinforcement can be mobilised — the section is too small or the concrete too weak, and **adding links cannot resolve it**. The only remedies are a larger section, a higher concrete grade, or reduced demand.

C = 1/(A+B)	Discriminant	Outcome
C > 2	Positive	Real solution, θ < 45° — section can be designed
C = 2	Zero	Boundary case, θ = 45° exactly
C < 2	Negative	No real angle, θ > 45° — section FAILS (crushing)

7. Implementation Mapping

The derived quantities map onto the code variables and spreadsheet columns as follows. Forces from the analysis package are taken in absolute value (sign denotes direction only).

Derived term	Formula	Code variable	Excel
A (shear)	$V_{Ed} /(\alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd})$	A_Major / A_Minor	—
B (torsion)	$T_{Ed} /(2 \cdot v_1 \cdot \alpha_{cw} \cdot f_{cd} \cdot A_k \cdot t_{ef})$	B_Major / B_Minor	—
C	1 /(A + B)	_1_A_plus_B_*	Col C
Root 1 / 2	$(C \pm \sqrt{(C^2-4)})/2 \rightarrow$ atan	Solution1_* / Solution2_*	Sol 1 / Sol 2
Min of pair	min(Sol1, Sol2)	Min_Solution1_Solution2_*	—
Design angle	max(Major, Minor), floor 22°	Design_Angle	—
Feasibility	$C \geq 2 \Leftrightarrow$ discriminant ≥ 0	discriminant < 0 → 9999	C ≥ 2 check
Verdict	$\theta \leq 45° \rightarrow$ pass	check2_…_sufficient	Verdict

Unit handling: V_{Ed} is entered in kN and multiplied by 10^3 to N; T_{Ed} is entered in kNm and multiplied by 10^6 to N·mm; all lengths are in mm and f_{cd} in N/mm². These conversions are embedded in the A and B expressions.

8. Assumptions and Limitations

- α_{cw} is taken as 1.0 (non-prestressed members). Prestressed sections require the appropriate α_{cw} per EC2.
- $f_{cd} = f_{ck} / 1.5$ assumes $\alpha_{cc} = 1.0$; confirm against the relevant National Annex (the UK NA adopts $\alpha_{cc} = 0.85$ in some contexts).
- f_{ck} is capped at 50 N/mm²; sections using concrete above C50 require review of the high-strength provisions.
- The shear-induced additional longitudinal tension (ΔF_{td}) is treated separately and is not part of this strut-angle derivation.
- Lower strut-angle limit $\cot\theta \leq 2.5$ ($\theta \geq 21.8°$) is applied by flooring the adopted angle at 22°.
- Axial force is not credited in the crushing check; this is conservative.

End of technical note. Prepared as internal software reference documentation. Verify all results against the governing edition of EC2 and the project National Annex before use in design.